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DESCRIPTION OF A FLOW OF A GAS-CONDENSATE MIXTURE IN AN AXISYMMETRIC CAPILLARY TUBE BY THE DENSITY-FUNCTIONAL METHOD

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An isothermal flow of a two-phase multicomponent mixture through a small-diameter capillary tube is examined by the density-functional method. For low ratios of the characteristic radius of the capillary to its length, a general form of the dominating term in the asymptotic solution is found. An improved version of the law of mixture transfer is obtained. The form of possible corrections to the Darcy law for the filtration rates of the phases is discussed.

Key words*: multicomponent two-phase mixture, capillary tube, asymptotic solution, Lagrange multipliers.*

For gas-condensate and oil-and-gas fields, a situation is typical in which the native hydrocarbon mixture undergoes a liquid–gas phase transition. During field development, the gas and the liquid move in the pore space of the rock, interacting with the rock and with each other owing to hydrodynamic viscous-friction forces and owing to the exchange by mixture components during the phase transition. The macroscopic description of the flow of a two-phase mixture in a porous medium obeys the Darcy filtration law [1]

$$u^a_{\rm g} = -\frac{k_{\rm g}(s)}{\mu_{\rm g}}(\partial_a p_{\rm g} + \rho_{\rm g}\partial_a\varphi), \qquad u^a_{\rm liq} = -\frac{k_{\rm liq}(s)}{\mu_{\rm liq}}(\partial_a p_{\rm liq} + \rho_{\rm liq}\partial_a\varphi), \tag{1}$$

where $u^a_{\rm g}$ and $u^a_{\rm liq}$ are the filtration rates of the gas and hydrocarbon liquid (condensate or oil), $p_{\rm g}$ and $p_{\rm liq}$ are the gas pressure and liquid pressure, $\rho_{\rm g}$ and $\rho_{\rm liq}$ are the mass densities of the gas and liquid, $\mu_{\rm g}$ and $\mu_{\rm liq}$ are the shear viscosities of the gas and liquid, $k_{\rm g}(s)$ and $k_{\rm liq}(s)$ are the phase permeabilities of the rock, which depend on the saturation degree of the liquid s, and φ is the gravity potential.

The classical filtration law (1) takes into account the friction between the moving phases and the rock. It is of interest to improve the filtration law by making allowance for possible cross-terms involving pressure gradients in the phases and standing to describe the friction between the moving phases and the inertial effects.

In the present work, we consider the problem of a steady-state isothermal flow of a multicomponent two-phase mixture in an axisymmetric capillary tube. The hydrodynamic description of the system is based on the density-functional method [2–5], which needs no *a priory* setting of the spatial positions of interfaces between the phases. For the flow in a small-diameter axisymmetric capillary, the dominating term of the asymptotic solution in the form of a series expansion in powers of the characteristic tube radius-to-length ratio is found. Based on the data obtained, we discuss the form of possible corrections in expressions (1) for the filtration rates of the phases.

We are going first to reproduce the fundamental equations used to describe a multicomponent mixture within the density-functional approach in an isothermal case [2]. The gravity forces are assumed to be negligibly weak.

Let the multicomponent mixture occupy a certain domain D in space with a smooth boundary ∂D. The subscripts i, j, and k refer to component numbers and take the values $1, \ldots, M$ (M is the total number of all components in the mixture). The subscripts a, b, and c refer to Cartesian coordinates x^a. Summation over repeated indices is implied.

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We consider only isothermal processes and introduce the following notations: n_i and m_i are the molar density and molar mass of the ith component, $\rho = m_i n_i$ is the mass density of the mixture, I_{ia} is the flux of the ith component, $v_a = \rho^{-1} m_i I_{ia}$ is the mean-mass velocity of the mixture, $Q_{ia} = I_{ia} - n_i v_a$ is the diffusion flux, p_{ab} is the symmetrical strain tensor of the mixture, $\partial_t = \partial/\partial t$ and $\partial_a = \partial/\partial x^a$ are derivatives with respect to time and coordinate, $g_{,i} = \partial g/\partial n_i$, and g is the density function of the components.

The flow of the mixture is defined by the conservation equations for the mixture components and momentum [6]:

$$\partial_t n_i + \partial_a I_{ia} = 0; \tag{2}$$

$$\rho(\partial_t v_a + v_b \partial_b v_a) = \partial_b p_{ab}. \tag{3}$$

The boundary conditions have the form

$$v_a\Big|_{\partial D} = 0, \qquad l_a Q_{ia}\Big|_{\partial D} = 0, \tag{4}$$

where l_a is the internal normal to the surface ∂D.

To close problem (2)–(4), we need explicit expressions for the diffusion fluxes Q_{ia} and stress tensor p_{ab} of the mixture. To derive these expressions, we use the free energy of the mixture, defined as a density functional [2]

$$F = \int\limits_D \theta_v \, dV + \int\limits_{\partial D} \theta_s \, dA. \tag{5}$$

Here $\theta_v = (1/2)\nu_{ij}\, \partial_a n_i \, \partial_a n_j + f$, $f(n_i)$ is the free energy on the homogeneous state in unit volume, $\theta_s(n_i)$ is the surface energy of the interface between the mixture and the solid phase (surface tension), and ν_{ij} is a positively defined symmetrical matrix whose elements, for simplicity, are assumed to be independent of the densities n_i.

The representation of the free energy as a functional of form (5) was first proposed in [7] and subsequently used in treating many problems [8, 9].

It was shown [2] that, in a static case, the theory based on expression (5) admits representation of equilibrium-state parameters of a multiphase system as continuous functions. Introducing squared density gradients into expression (5), one can consider some interface regions with continuously varying problem parameters instead of geometric surfaces between dissimilar phases.

In the static case, the equilibrium states of the mixture correspond to critical points of functional (5) with preset amounts of each component in the mixture [2]:

$$N_i = \int\limits_D n_i \, dV. \tag{6}$$

Hence, the necessary condition of equilibrium can be written as a variational equation

$$\delta F - \lambda_i \, \delta N_i = 0, \tag{7}$$

where λ_i are the Lagrange multipliers. Calculating the variation of functional (5) explicitly, we obtain

$$\delta F = \int\limits_D \Phi_i \, \delta n_i \, dV + \int\limits_{\partial D} \psi_i \, \delta n_i \, dA,$$

$$\Phi_i = f_{,i} - \nu_{ij}\, \Delta n_j, \quad \psi_i = \theta_{s,i} - \nu_{ij} l_a \, \partial_a n_j, \quad \Delta = \partial_a \, \partial_a. \tag{8}$$

In this way, the variational equations (7) are reduced to an elliptical system of differential equations

$$\Phi_i - \lambda_i = 0 \tag{9}$$

with boundary conditions

$$\psi_i = 0. \tag{10}$$

The unknown Lagrange multipliers λ_i are to be found from known values of N_i [see (6)]. Since the value of $f_{,i}$ equals the chemical potential of the ith component, the Lagrange multipliers for homogeneous states equal the chemical potentials of the mixture components.

Returning to the dynamic equations, we assume that, in the dynamic case, the boundary conditions (10) also hold.

Next, we introduce into the present consideration the kinetic energy of the mixture

$$K = \frac{1}{2} \int\limits_D \rho v_a v_a \, dV.$$

Using (2)–(4), (8), and (10), we can easily calculate the derivative of the total energy of the mixture $E = K + F$:

$$\frac{dE}{dt} = \int\limits_D (-\tau_{ab} \, \partial_a v_b + Q_{ia} \, \partial_a \Phi_i) \, dV;$$

$$\tau_{ab} = p_{ab} - \sigma_{ab}, \qquad \sigma_{ab} = -\nu_{ij} \, \partial_a n_i \, \partial_b n_j + \delta_{ab}(\theta_v - \Phi_i n_i). \tag{11}$$

The tensors σ_{ab} and τ_{ab} are interpreted as the static-strain tensor and the viscous-stress tensor, respectively. In other words, the matrix σ_{ab} describes stresses in a spatially nonuniform mixture, independent of flow parameters. In a spatially uniform medium, the matrix σ_{ab} reduces to ordinary stresses in an ideal liquid $\sigma_{ab} = -p\delta_{ab}$ ($p = f_{,i} n_i - f$ is the hydrostatic pressure).

The expressions for the viscous-stress tensor τ_{ab} of the diffusion fluxes Q_{ia} should be consistent with the condition of dissipativity of the model $dE/dt \leqslant 0$. The simplest (but nonunique) way to meet this condition is to use the Navier–Stokes model for viscous stresses and the generalized Fick law for diffusion:

$$\tau_{ab} = (\zeta - 2\mu/3)\delta_{ab} \, \partial_c v_c + \mu(\partial_a v_b + \partial_b v_a); \tag{12}$$

$$Q_{ia} = -D_{ij} \, \partial_a \Phi_j. \tag{13}$$

Here $\zeta(n_i)$ and $\mu(n_i)$ are the positive coefficients of volume and shear viscosities of the mixture, $D_{ij}(n_k)$ is the symmetrical matrix of diffusion coefficients with nonnegative eigenvalues (the only zero eigenvalue of this matrix corresponds to the eigenvector m_i).

Let us verify the consistency between the static equations (9) and the hydrodynamic equations (2), (3), (12), and (13). We can conveniently use the following, easy-to-check equality:

$$\partial_b \sigma_{ab} = -n_i \, \partial_a \Phi_i. \tag{14}$$

Let conditions (9) be fulfilled and the mean-mass velocity be zero. Substitution of relations (9) into (13) shows that the diffusion fluxes are zero and the conservation equations for components (2) are satisfied. In this case, the momentum equations (3) are also satisfied by virtue of identity (14).

Vice versa, let the mean-mass velocity and the diffusion fluxes be zero. Then, Eq. (13) yields the equality $\Phi_i = \Phi_{i0} + m_i \psi$, where Φ_{i0} are constants and ψ is some function of the coordinates. It follows from the condition of equilibrium $\partial_b \sigma_{ab} = 0$ and equality (14) that ψ is a constant. Thus, the validity of (9) is proved.

Let us examine now the flow of a multicomponent mixture through an axisymmerical capillary tube of variable radius. We use the following cylindrical coordinates: z is the coordinate along the capillary axis, r is the distance from the axis, and θ is the azimuthal angle. The capillary radius is defined as a function $r = r_c(z)$. The unknown quantities in this problem are the density fields $n_i(z, r)$ and the velocity fields $v_z(z, r)$ and $v_r(z, r)$.

The hydrodynamic equations (2) and (3) reduce to the system of equations

$$\partial_z(n_i v_z + Q_{iz}) + (\partial_r + r^{-1})(n_i v_r + Q_{ir}) = 0; \tag{15}$$

$$\begin{gathered} \rho(v_z \partial_z + v_r \partial_r) v_z = \partial_z p_{zz} + (\partial_r + r^{-1}) p_{zr}, \\ \rho(v_z \partial_z + v_r \partial_r) v_r = \partial_z p_{zr} + (\partial_r + r^{-1}) p_{rr} - r^{-3} p_{\theta\theta}, \end{gathered} \tag{16}$$

in which, in accordance with relations (11)–(13), the expressions for the diffusion fluxes of stress-tensor components have the form

$$Q_{iz} = -D_{ij} \, \partial_z \Phi_j, \qquad Q_{ir} = -D_{ij} \, \partial_r \Phi_j,$$

$$p_{zz} = -\nu_{ij} \, \partial_z n_i \, \partial_z n_j + \theta_v - \Phi_i n_i + (\zeta + 4\mu/3)\partial_z v_z + (\zeta - 2\mu/3)(\partial_r v_r + r^{-1} v_r),$$

$$p_{rr} = -\nu_{ij} \, \partial_r n_i \, \partial_r n_j + \theta_v - \Phi_i n_i + (\zeta - 2\mu/3)(\partial_z v_z + r^{-1} v_r) + (\zeta + 4\mu/3)\partial_r v_r,$$

$$p_{\theta\theta} = r^2(\theta_v - \Phi_i n_i) + (\zeta - 2\mu/3)r^2(\partial_z v_z + \partial_r v_r) + (\zeta + 4\mu/3) r v_r,$$

$$p_{zr} = p_{rz} = -\nu_{ij}\,\partial_r n_i\,\partial_z n_j + \mu(\partial_z v_r + \partial_r v_z),$$

$$\theta_v = \nu_{ij}(\partial_z n_i \partial_z n_j + \partial_r n_i \partial_r n_j)/2 + f, \quad \Phi_i = f_{,i} - \nu_{ij}(\partial_z^2 n_j + r^{-1}\partial_r(r\,\partial_r n_j)).$$

The boundary conditions on the capillary walls $r = r_c(z)$ follow from relations (4) and (10):

$$v_r = 0, \quad v_z = 0, \quad Q_{ir} = 0, \quad \nu_{ij}\,\partial_r n_j = \theta_{s,i}. \tag{17}$$

The boundary conditions at the capillary axis ($r = 0$) follow from the symmetry of the problem and flow continuity:

$$v_r = 0, \quad \partial_r v_z = 0, \quad Q_{ir} = 0, \quad \partial_r n_i = 0. \tag{18}$$

Equation (15) yields the ordinary continuity equation

$$\partial_z(\rho v_z) + (\partial_r + r^{-1})(\rho v_r) = 0, \tag{19}$$

and also the integral conservation laws

$$\frac{dq_i}{dz} = 0, \qquad q_i = 2\pi \int\limits_0^{r_c} (n_i v_z + Q_{iz}) r\,dr. \tag{20}$$

Here q_i is the total flux of the ith component through the capillary tube.

In addition to the boundary conditions (7) and (18), boundary conditions at the ends of the capillary tube need to be formulated. These conditions will be given below, when constructing the solution of interest.

We assume that the coordinate z varies from 0 to $L > 0$ and R_c is the characteristic radius of the capillary tube. We can conveniently introduce the small parameter $\delta = R_c/L$ and the dimensionless coordinates $x = z/L$ and $y = r/R_c$. We also introduce the dimensionless capillary radius $R(x) = R_c^{-1} r_c(z)$. For simplicity, we use the system of units in which $L = 1$.

To construct the asymptotic solution containing the small parameter δ, we have to set the order of the sought quantities with respect to δ. Here, the governing equations and the boundary conditions, which relate the sought quantities, should be fulfilled. Physically, the sought asymptotics describes the flow in a fixed-length capillary of infinitely small radius. In this case, we have to choose the order of the parameters ν_{ij} and $\theta_{s,i}$, which determine the thickness of surface layers and interphase zones [2, 3], and also the order of diffusivities. We use the expressions

$$\nu_{ij} = \delta^2 \alpha_{ij}, \quad \theta_{s,i} = \delta \beta_i, \quad D_{ij} = \delta^2 d_{ij}(n_k), \tag{21}$$

where α_{ij} and β_i are some finite constants. We represents the sought quantities as series:

$$n_i = \sum_{\alpha=0}^{+\infty} \delta^\alpha n_i^\alpha(x,y), \quad v_z = \delta^2 \sum_{\alpha=0}^{+\infty} \delta^\alpha v_z^\alpha(x,y), \quad v_r = \delta^2 \sum_{\alpha=0}^{+\infty} \delta^\alpha v_r^\alpha(x,y). \tag{22}$$

Substitution of relations (21) and (22) into the boundary conditions (17) and (18) defines the boundary conditions for the dominating terms in the asymptotic solution:

— for $y = R(x)$,

$$v_r^0 = 0, \qquad v_z^0 = 0, \tag{23}$$

$$d_{ij}(n_k^0)\,\partial_y \Phi_j^0 = 0, \tag{24}$$

$$\alpha_{ij}\,\partial_y n_j^0 = \beta_i; \tag{25}$$

— for $y = 0$,

$$v_r^0 = 0, \qquad \partial_y v_z^0 = 0, \tag{26}$$

$$d_{ij}(n_k^0)\,\partial_y \Phi_j^0 = 0, \tag{27}$$

$$\partial_y n_i^0 = 0. \tag{28}$$

Here $\Phi_i^0 = f_{,i}(n_k^0) - \alpha_{ij} y^{-1}\,\partial_y(y \partial_y n_j^0)$.

In the major approximation, Eq. (19) yields the ordinary differential equation $(\partial_y + y^{-1})(m_i n_i^0 v_r^0) = 0$, whose integral has the form $m_i n_i^0 v_r^0 = y^{-1} C_1(x)$. The latter expression is consistent with the boundary conditions (23) and (26) only if $C_1(x) = 0$. Thus, we have

$$v_r^0 = 0. \tag{29}$$

Invoking (14) and (29), from (15) and (16) we obtain the following equations:

$$(\partial_y + y^{-1})(d_{ij}(n_k^0)\, \partial_y \Phi_j^0) = 0; \tag{30}$$

$$-n_i^0\, \partial_z \Phi_i^0 + (\partial_y + y^{-1})(\mu(n_k^0)\, \partial_y v_z^0) = 0; \tag{31}$$

$$-n_i^0\, \partial_y \Phi_i^0 = 0. \tag{32}$$

Relations (24), (27), (30), and (32) are valid if the values of Φ_i^0 do not depend on the radial coordinate y, i.e., if the following equations hold:

$$\Phi_i^0 = \lambda_i(x). \tag{33}$$

A comparison of system (33) with the equilibrium conditions (9) makes it possible to interpret (33) as a condition for an equilibrium distribution of the mixture components in the capillary cross section. Thus, system (33) with the boundary conditions (25) and (28) defines the densities of the mixture components $n_i^0 = n_i^0(x, y)$. In fact, the dependence of n_i^0 on the longitudinal coordinate x can be expressed through the dependence of the parameters λ_i and capillary radius R on x. Therefore, the densities of the mixture components can be represented as the functions $n_i^0 = n_i^0(y; R, \lambda_j)$. The parameters $\lambda_i(x)$ should be considered as local chemical potentials of the mixture.

The particular functional form of the solution $n_i^0 = n_i^0(y; R, \lambda_j)$ of problem (33) with the boundary conditions (25) and (28) is defined by the used free-energy function of the homogeneous state $f(n_i)$. Usually, the description of real hydrocarbon mixtures employs semi-empirical cubic equations of state [10, 11]. In this case, the analytical expressions for the function $f(n_i)$ turn out to be too complex, thus, excluding the possibility of calculating the functions $n_i^0(y; R, \lambda_j)$ in a final analytical form. Therefore, generally, to solve problem (33), (25), and (28), one has to employ some numerical algorithms. Yet, as it will be shown below, an analytical consideration of the problem of the flow of a gas-condensate mixture in an axisymmetric capillary tube yields several results of utmost importance.

We introduce the functions $J_i(y; R, \lambda_j)$ defined by the equation

$$y^{-1}\, \partial_y (y \mu^0\, \partial_y J_i) = -n_i^0 \tag{34}$$

and the boundary conditions

$$J_i|_{y=R} = 0, \qquad \partial_y J_i|_{y=0} = 0,$$

where $\mu^0(y; R, \lambda_j) = \mu(n_i^0(y; R, \lambda_j))$. The functions $J_i(y; R, \lambda_j)$ can be found as double integrals:

$$J_i(y; R, \lambda_j) = \int_y^R (y_2 \mu^0(y_2; R, \lambda_j))^{-1} \int_0^{y_2} y_1 n_i^0(y_1; R, \lambda_j)\, dy_1\, dy_2.$$

A comparison between Eqs. (31) and (34) readily gives the following expression for the longitudinal velocity of the flow: $v_z^0(x, y) = -J_i\, d\lambda_i/dx$. Substitution of this expression into (20) yields the major asymptotics of the fluxes of the mixture components in the form $q_i = \delta^2 q_i^0$, where

$$q_i^0 = -\Psi_{ij} \frac{d\lambda_j}{dx}, \quad \Psi_{ij}(y; R, \lambda_k) = 2\pi \int_0^R (n_i^0 J_j + D_{ij}(n_k^0)) y\, dy. \tag{35}$$

The matrix Ψ_{ij} is symmetric and positively defined:

$$\Psi_{ij} = 2\pi \int_0^R \Big[(y \mu^0(y; R, \lambda_k))^{-1} \int_0^y y_1 n_i^0(y_1; R, \lambda_k)\, dy_1 \int_0^y y_2 n_j^0(y_2; R, \lambda_k)\, dy_2 + d_{ij}(n_k^0(y; R, \lambda_k)) \Big] y\, dy. \tag{36}$$

Thus, the transfer law (35) satisfies the Onsager theory.

The equations of conservation of the mixture components $dq_i^0/dx = 0$ and the transfer law (35) form a system of second-order ordinary differential equations for the chemical potentials of the mixture $\lambda_i(x)$ along the capillary. The values of λ_i at the ends of the capillary can be used as boundary conditions.

Transfer law (35) is a general one irrespective to whether the mixture is in a single-phase or in a two-phase state. If the mixture is in a two-phase state and the zone of the transition between the phases is narrow compared to the capillary radius (so that the radius $y = \tau$ for the interface between the phases can be introduced), then a more accurate formulation of law (35) in terms of the phase filtration velocities is possible. Let the liquid phase be a wetting one (i. e., the liquid resides near the capillary walls), n_{ig}^0 and n_{il}^0 be the densities of the components in the gas and in the liquid, and $d_{ijg} = d_{ij}(n_{kg}^0)$ and $d_{ijl} = d_{ij}(n_{kl}^0)$ be the reduced diffusivities in the gas and in the liquid. Dividing the integration domain in expression (36) into subregions occupied by the gas and by the liquid and using the Gibbs–Duhem relations $dp_{\rm g} = n_{ig}^0\, d\lambda_i$ and $dp_{\rm liq} = n_{il}^0\, d\lambda_i$, we bring law (35) to the form

$$q_i^0 = q_{ig} + q_{il}, \quad q_{ig} = \pi R^2 \Big(n_{ig}^0 u_{\rm g} - \Big(\frac{\tau}{R}\Big)^2 d_{ijg}\, \frac{d\lambda_j}{dx} \Big), \quad q_{il} = \pi R^2 \Big(n_{il}^0 u_{\rm liq} - \Big(1 - \Big(\frac{\tau}{R}\Big)^2\Big) d_{ijl}\, \frac{d\lambda_j}{dx} \Big);$$

$$u_{\rm g} = -R^2 \Big(\frac{\Sigma_1}{\mu_{\rm g}} + \frac{\Sigma_2}{\mu_{\rm liq}} \Big) \frac{dp_{\rm g}}{dx} - \frac{R^2 \Sigma_3}{\mu_{\rm liq}}\, \frac{dp_{\rm liq}}{dx}, \quad u_{\rm liq} = -\frac{R^2 \Sigma_4}{\mu_{\rm liq}}\, \frac{dp_{\rm g}}{dx} - \frac{R^2 \Sigma_5}{\mu_{\rm liq}}\, \frac{dp_{\rm liq}}{dx}; \qquad (37)$$

$$\Sigma_1 = \frac{1}{8}\Big(\frac{\tau}{R}\Big)^4, \quad \Sigma_2 = -\frac{1}{2}\Big(\frac{\tau}{R}\Big)^4 \ln\frac{\tau}{R}, \quad \Sigma_3 = \Sigma_4 = \frac{1}{4}\Big(\frac{\tau}{R}\Big)^2 \Big(1 - \Big(\frac{\tau}{R}\Big)^2 + 2\Big(\frac{\tau}{R}\Big)^2 \ln\frac{\tau}{R}\Big),$$

$$\Sigma_5 = \frac{1}{8}\Big(1 - \Big(\frac{\tau}{R}\Big)^2\Big)^2 - \frac{1}{4}\Big(\frac{\tau}{R}\Big)^2 \Big(1 - \Big(\frac{\tau}{R}\Big)^2 + 2\Big(\frac{\tau}{R}\Big)^2 \ln\frac{\tau}{R}\Big).$$

Expressions (37) give a generalization of expressions (1) for the filtration rates of the phases. It should be noted that these expressions contain cross-terms, i. e., the gas filtration rate depends on the pressure gradient in the liquid, and vice versa, the liquid filtration rate depends on the pressure gradient in the gas. The quantities $R^2\Sigma_A$ have the meaning of phase permeabilities. With the mutual friction of the phases taken into account, we have here four independent coefficients instead of the two coefficients in the classical law (1). For real porous media, the analytical expressions for the phase permeabilities are more evolved and depend on particular properties of the rock. Nevertheless, the functional form of the filtration law should correspond to expressions (37).

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REFERENCES

1. V. N. Nikolaevskii et al., *Flow of Hydrocarbon Mixtures in Porous Media* [in Russian], Nedra, Moscow (1968).
2. O. Yu. Dinariev, “Hydrodynamic description of a multicomponent multiphase mixture in narrow pores and thin layers,” *Prikl. Mat. Mekh.*, **59**, No. 5, 776–783 (1995).
3. O. Yu. Dinariev, “Flow in a capillary tube with surface-active walls. Method of density functional,” *Izv. Ross. Akad. Nauk, Mekh. Zhidk. Gaza*, No. 2, 141–148 (1997).
4. O. Yu. Dinariev, “Temperature effects in the description of a multicomponent mixture by the density-functional method,” *Prikl. Mat. Mekh.*, **62**, No. 3, 433–442 (1998).
5. O. Yu. Dinariev, “Description of capillary-gravity waves in the density-functional theory,” *Izv. Ross. Akad. Nauk, Mekh. Zhidk. Gaza*, No. 5, 145–151 (1999).
6. L. I. Sedov, *Mechanics of Continuous Media* [in Russian], Vol. 1, Nauka, Moscow (1973).
7. J. W. Cahn and J. E. Hilliard, “Free energy of a nonuniform system. 1. Interfacial free energy,” *J. Chem. Phys.*, **28**, 258–267 (1958).
8. M. E. Eglit, “Generalization of the ideal incompressible liquid model,” *Prikl. Mat. Mekh.*, **29**, No. 2, 351–354 (1965).
9. L. M. Truskinovskii, “Equilibrium interfaces between phases,” *Dokl. Akad. Nauk SSSR*, **265**, No. 2, 306–310 (1982).
10. R. C. Reid, J. M. Prausnitz, and T. Sherwood, *The Properties of Gases and Liquids*, McGraw-Hill, New York (1977).
11. G. R. Gurevich and A. I. Brusilovskii, *Calculation of the Phase State and Properties of Gas-Condensate Mixtures: Reference Manual* [in Russian], Nedra, Moscow (1984).